Capillary waves

$$\frac{\partial \phi^2}{\partial x^2} + \frac{\partial \phi^2}{\partial z^2} = 0$$

$$\begin{cases} \left.\frac{\partial \phi}{\partial z}\right|_{z=-H} = 0 \\ \left.\frac{\partial \phi}{\partial z}\right|_{z=0} = \frac{\partial \eta}{\partial t} \\ \left.\frac{\partial \phi}{\partial t}\right|_{z=0} + g\eta = \frac{\sigma}{\rho}\frac{\partial \eta^2}{\partial x^2} \end{cases}$$

$$p_{atm} - p = \frac{\sigma}{r} = \approx \sigma \frac{\partial \eta^2}{\partial x^2}$$

$$\begin{cases} \eta(x,t) = a\cos(kx - \omega t) \\ u = a\omega \frac{\operatorname{ch} k(z+H)}{\operatorname{sh} kH}\cos(kx-\omega t) \\ w = a\omega \frac{\operatorname{sh} k(z+H)}{\operatorname{sh} kH}\sin(kx-\omega t) \\ \omega = \sqrt{k\left(g + \frac{\sigma k^2}{\rho}\right)\operatorname{th} kH} \end{cases}$$

$$c = \sqrt{\left(\frac{g}{k} + \frac{\sigma k}{\rho}\right) \operatorname{th} kH} = \sqrt{\left(\frac{g\lambda}{2\pi} + \frac{2\pi\sigma}{\rho\lambda}\right) \operatorname{th} \frac{2\pi H}{\lambda}}$$