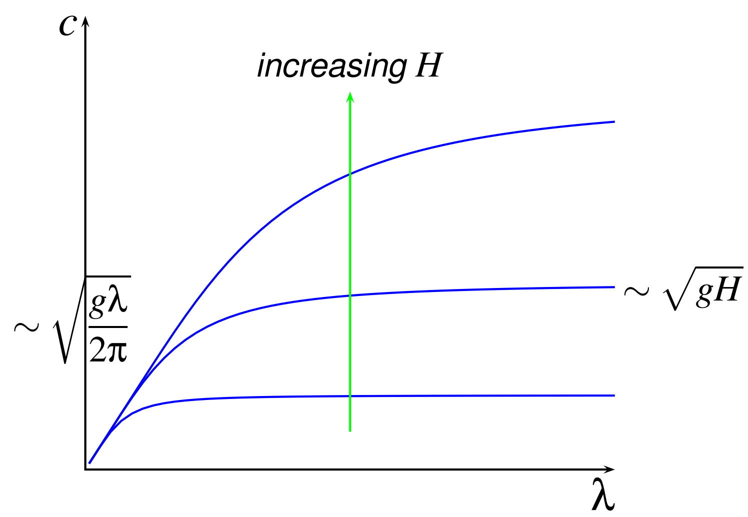
Surface gravity waves

$$\frac{\partial\phi^2}{\partial x^2}+\frac{\partial\phi^2}{\partial z^2}=0$$

$$\begin{cases} \left.\dfrac{\partial\phi}{\partial z}\right|_{z=-H}=0 \\ \left.\dfrac{\partial\phi}{\partial z}\right|_{z=0}=\dfrac{\partial\eta}{\partial t} \\ \left.\dfrac{\partial\phi}{\partial t}\right|_{z=0}+g\eta=0 \end{cases}$$

$$\begin{cases} \eta(x,t)=a\cos(kx-\omega t) \\ u=a\omega\dfrac{\operatorname{ch}k(z+H)}{\operatorname{sh}kH}\cos(kx-\omega t) \\ w=a\omega\dfrac{\operatorname{sh}k(z+H)}{\operatorname{sh}kH}\sin(kx-\omega t) \\ \omega=\sqrt{gk\operatorname{th}kH} \end{cases}$$

- $c = \frac{\omega}{k} = \sqrt{\frac{g}{k}\,\mathrm{th}\,kH} = \sqrt{\frac{g\lambda}{2\pi}\,\mathrm{th}\,\frac{2\pi H}{\lambda}}$
- $p' = \rho g a \frac{\mathrm{ch}\,k(z+H)}{\mathrm{ch}\,kH}\cos(kx-\omega t)$
- $c_g = \frac{d\omega}{dk} = \frac{c}{2}\left(1+\frac{2kH}{\mathrm{sh}\,2kH}\right)$



$$E_k = \frac{1}{\lambda}\int_0^\lambda \int_{-H}^{\eta} \frac{1}{2}\rho(u^2+w^2)\,dz\,dx \approx \frac{1}{\lambda}\int_0^\lambda \int_{-H}^{0} \frac{1}{2}\rho(u^2+w^2)\,dz\,dx = \frac{1}{4}\rho g a^2$$

$$E_p = \frac{1}{\lambda}\int_0^\lambda \left[\int_{-H}^{\eta}\rho g z\,dz - \int_{-H}^{0}\rho g z\,dz\right]dx = \frac{1}{\lambda}\int_0^\lambda \int_0^{\eta}\rho g z\,dz\,dx = \frac{1}{2}\rho g\overline{\eta^2} = \frac{1}{4}\rho g a^2$$