Prandtl-Kolmogorov

$$\nu_e = c\, k^{1/2}\, \ell_m, \qquad \frac{\bar{D}k}{Dt} = -\nabla\cdot\mathbf{T}^\star + \mathcal{P} - \varepsilon$$

- $T_j^\star \equiv \frac{1}{\rho}\overline{p' u'_j} + \frac{1}{2}(\overline{u'_i u'_i u'_j}) - 2\nu\overline{u'_i e'_{ij}} = -\nu_k \frac{\partial k}{\partial x_j}$
- $\mathcal{P} \equiv -\overline{u'_i u'_j}\,\frac{\partial \bar{u}_i}{\partial x_j} = \left(-\frac{2}{3}k\delta_{ij} + 2\nu_e\,\bar{e}_{ij}\right)\frac{\partial \bar{u}_i}{\partial x_j} = 2\nu_e\,\bar{e}_{ij}\bar{e}_{ij}$
- $\varepsilon \equiv 2\nu\overline{e'_{ij}e'_{ij}} = C_D\frac{k^{3/2}}{\ell_m} \geq 0$

$$\frac{\bar{D}k}{Dt} = \nabla\cdot(\nu_k\nabla k) + 2\nu_e\,\bar{\mathbf{e}}:\bar{\mathbf{e}} - \varepsilon$$

In a boundary layer :

$$2\nu_e \, \bar{\mathbf{e}} : \bar{\mathbf{e}} \approx \varepsilon$$

$$\nu_e \left(\frac{dU}{dy}\right)^2 = c\, k^{1/2}\, \ell_m \left(\frac{dU}{dy}\right)^2 \approx C_D \frac{k^{3/2}}{\ell_m}$$

$$\Rightarrow k = \frac{c}{C_D} \ell_m^2 \left(\frac{dU}{dy}\right)^2, \qquad \nu_e = \sqrt{\frac{c^3}{C_D}} \ell_m^2 \left|\frac{dU}{dy}\right| = \ell_m^2 \left|\frac{dU}{dy}\right|$$