Boussinesq-Prandtl

$$-\overline{u_i'u_j'} = -\frac{2}{3}k\delta_{ij} + 2\nu_e\bar{e}_{ij}$$

$$\nu_e = u'\ell_m$$

- $\bar{u}(y+\ell_m) \approx \bar{u}(y) + \frac{d\bar{u}}{dy}\ell_m : u' \sim \ell_m\left|\frac{d\bar{u}}{dy}\right|$
- $\ell_m \sim \kappa\, y.$

$$\frac{1}{\rho}\tau_{ij}^T = -\frac{\bar{p}}{\rho}\,\delta_{ij} + 2\nu\bar{e}_{ij} - \overline{u_i'u_j'}$$

$$= -\left(\frac{\bar{p}}{\rho} + \frac{2k}{3}\right)\,\delta_{ij} + 2(\nu+\nu_e)\,\bar{e}_{ij}$$

$$\nu_e = \kappa^2 y^2\left|\frac{d\bar{u}}{dy}\right|$$

$$-\overline{u'v'} = \nu_e\frac{\partial\bar{u}}{\partial y} = \kappa^2y^2\left|\frac{d\bar{u}}{dy}\right|\frac{d\bar{u}}{dy} \approx const. = \frac{\tau_0}{\rho} = u_\star^2$$

$$\Rightarrow \qquad \frac{\bar{u}}{u_\star} = \frac{1}{\kappa}\ln y + const.$$