

Vorticity equation

$$\frac{\partial \mathbf{u}}{\partial t} - \mathbf{u} \times \boldsymbol{\omega} + \nabla \left(\frac{1}{2} u^2 \right) = \mathbf{f} - \frac{1}{\rho} \nabla p + \frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma}$$

$$\frac{\partial \boldsymbol{\omega}}{\partial t} + \boldsymbol{\omega} (\nabla \cdot \mathbf{u}) - (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + (\mathbf{u} \cdot \nabla) \boldsymbol{\omega} = \nabla \times \mathbf{f} - \nabla \left(\frac{1}{\rho} \right) \times \nabla p + \nabla \times \left(\frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma} \right)$$

$$\rho \frac{D}{Dt} \left(\frac{\boldsymbol{\omega}}{\rho} \right) = (\boldsymbol{\omega} \cdot \nabla) \mathbf{u} + \nabla \times \mathbf{f} + \frac{1}{\rho^2} \nabla \rho \times \nabla p + \nabla \times \left(\frac{1}{\rho} \nabla \cdot \boldsymbol{\sigma} \right)$$

$$\nabla \times (\nabla \phi) = \mathbf{0}, \quad \nabla \times (\phi \mathbf{a}) = \phi \nabla \times \mathbf{a} + \nabla \phi \times \mathbf{a}, \quad \nabla \cdot (\nabla \times \mathbf{u}) = \nabla \cdot \boldsymbol{\omega} = 0$$

$$\nabla \times (\mathbf{a} \times \mathbf{b}) = \mathbf{a} (\nabla \cdot \mathbf{b}) - \mathbf{b} (\nabla \cdot \mathbf{a}) + (\mathbf{b} \cdot \nabla) \mathbf{a} - (\mathbf{a} \cdot \nabla) \mathbf{b}$$